A Mathematical Model of the Growth and Impact of the Nigeria's External Debt

[1]Tivde, T. [2]Ujah, A. E. and [3]Tivde, N. T.

[2,1]*Department of Mathematics/Statistics/Computer Science, federal University of Agriculture, Makurdi*
Corresponding Author: Tivde, T.

ABSTRACT: The study attempts to provide a simplified model of the formation of growth and impact of external debt burden of Federal Government of Nigeria, with respect to various factors including oil revenue and federal expenditures via a mathematical exploration of the relationships between various economic quantities.Our assumptions prior to the modelling process include a Keynesian economy as the economic system of consideration, oil revenue as sole source of governmental income, and linear dependence of the rate of change of external debt on the difference between federal revenue and expenditures. The study used Malthusian and Logistic growth model formation to determine the growth of external debt burden and economic theories to test the impact of external debt. It is revealed from the study that the debt profile is on the increase once more after the debt cancellation of 2005 by Paris Club of Creditors and instability with oil revenue. The recommendations are that the Nigeria government should ensure that debts incurred are channeled towards productive uses and urgently need to diversify the economy from oil.

KEY WORDS: Oil Revenue, Federal Expenditures Mathematical Model, Nigeria's External Debt,Keynesian Economy, Malthusian and Logistic growth models

I. INTRODUCTION

It of no doubt that external debts of Nigeria are gradually becoming huge and a central problem of Nigeria State. The issue of Nigeria's external debt has not generated much public concern prior to 1980. It was at the beginning of 1980s that it became a topical issue of discussion among populace not only to Nigeria but most third world and developing countries, they are faced with the scarcity of funds to finance major infrastructure projects in their countries thus, resulting to seek for funds from external sources to supplement their revenues from taxes and earning from other means, which are usually low when compared with other developed nations. As a result of this situation, most developing countries have a hype of debts which always server as barrier to economic growth and welfare of the citizens because the debts must be services and paid.

Since accumulating debts for the developing countries is situations they must leave with to achieve enhanced infrastructure facilities, most development economist are now advocating for a "favorable debts". A favorable debt is one whose the capital acquired has the potentials of high leftover after deducting the cost of loan.

The act of borrowing creates debts and this debt may be domestic or external. The focus of this study is on external debt which refers to that part of a nation's debt that is owed to creditors outside the nation. [1] defines external debt as that portion of a country's debt that is acquired from foreign sources such as foreign corporations, government or financial institutions. Acording to [external debt arises as a result of the gap between domestic savings and investment. As the gap widens, debt accumulates and this makes the country to continually borrow increasing amounts in order to stay afloat. He further defined Nigeria's external debt as the debt owed by the public and private sectors of the Nigerian economy to non-residents and citizens that is payable in foreign currency, goods and services.

Debt crisis occurs when a country has accumulated a huge amount of debt such that it can no longer effectively manage the debt which leads to several mishaps in the domestic political economy [2]. [14] defined debt crisis as a situation whereby a nation is severely indebted to external sources and is unable to repay the principal of the debt.

Sustainable economic growth is a major concern for any sovereign nation most especially the Less Developed Countries (LDCs) which are characterized by low capital formation due to low levels of domestic savings and investment [3]. It is expected that these LDC's when facing a scarcity of capital would resort to borrowing from external sources so as to supplement domestic saving ([5], [17], [18]).[20] asserted that countries borrow for two broad reasons; macroeconomic reason that is to finance higher level of consumption and investment or to finance transitory balance of payment deficit and avoid budget constraint so as to boost economic growth and reduce poverty. The constant need for governments to borrow in order to finance budget deficit has led to the creation of external debt.

External debt is a major source of public receipts and financing capital accumulation in any economy [3]. It is a medium used by countries to bridge their deficits and carry out economic projects that are able to increase the standard of living of the citizenry and promote sustainable growth and development. [stated that external borrowing ought to accelerate economic growth especially when domestic financing is inadequate. External debt also improves total factor productivity through an increase in output which in turn enhances Gross Domestic product (GDP) growth of a nation. The importance of external debt cannot be overemphasized as it is an ardent booster of growth and thus improves living standards thereby alleviating poverty.

It is widely recognized in the international community that excessive foreign indebtedness in most developing countries is a major impediment to their economic growth and stability [6]. Developing countries like Nigeria have often contracted large amount of external debts that has led to the mounting of trade debt arrears at highly concessional interest rates. [11] opined that accumulated debt service payments create a lot of problems for countries especially the developing nations reason being that a debt is actually serviced for more than the amount it was acquired and this slows down the growth process in such nations. The inability of the Nigerian economy to meet its debt service payments obligations has resulted in debt overhang or debt service burden that has militated against her growth and development [3].

The genesis of Nigeria's debt service burden dates back to 1978 after a fall in world oil prices. Prior to this occurrence Nigeria had incurred some minor debts from World Bank in 1958 with a loan of US$28million dollars for railway construction and the Paris Club debtor nations in 1964 from the Italian government with a loan of US$13.1 million for the construction of the Niger dam. The first major borrowing of US$1 billion known as the "Jumbo loan" was in 1978 from the International Capital Market (ICM) [4].

The unabated increase in the level of external debt service payments has led to huge imbalances in fiscal deficits and budgetary constraints that have militated against the growth of the Nigerian economy. The resultant effect of the debt quagmire in Nigeria could create some unfavourable circumstances such as crowding out of private investment, poor GDP growth e.t.c ([20]). In this situation, the debt will be financing economic growth, increasing the infrastructural capacity and expending output of the borrowing country.

[18] carried out a study on the effect of external debt on the economic growth of Nigeria. Annual time series data covering the period from 1970-2010 was used. The empirical analysis was carried out using econometric techniques of Ordinary least squares (OLS), Augmented Dickey-Fuller unit root test, Johansen Co-integration test and error correction method. The co-integration test shows long-run relationship amongst the variables and findings from the error correction model revealed that external debt has contribute positively to the growth of the Nigerian economy. In addition the study recommends that the Nigerian should ensure political and economic stability so as to ensure effective debt management.

It is observed that [10] investigated the impact of external debt on the economic growth of Tanzania using time series data on external debt and economic performance covering the period 1990-2010. It was observed through the Johansen co-integration test that no long-run relationship between external debt and GDP. However, the findings show that external debt and debt service both have significant impact on GDP growth with the total external debt stock having a positive effect of about 0.36939 and debt service payment having a negative effect of about 28.517. The study also identified the need for further research on the impact of external debt on foreign direct investments (FDIs) and domestic revenues [17] analyzed external debt and economic growth in Iran by observing the balance and long term relation of five variables (GDP, private investment, public investment, external debt and imports). Time series data covering the period 1974-2007 was used and the vector autoregressive model (VAR) technique of estimation was employed. Their findings revealed that external that has a negative effect on GDP and private investment and

pubic investment has a positive relationship with private investment.

In a similar pattern, [8] investigated the effect of external debt on the economic growth of eight selected heavily indebted African countries (Benin, Ethiopia, Mali, Madagascar, Mozambique, Senegal, Tanzania and Uganda) through the debt overhang and debt crowding out effect with ratio of external debt to gross national income as a proxy for debt overhang and debt service export ratio as a proxy for debt crowding out. Panel data covering the period 1991-2010 was used. The empirical investigation was carried out on a cross-sectional regression model with tests for stationarity using Augmented Dickey Fuller tests, heteroskedasticity and ordinary regression. The concluding result from estimation showed that external debt affects economic growth through debt crowding out rather than debt overhang.

There are various empirical studies that have been conducted to investigate the impact of external debt burden on economic growth in Nigeria and have arrived at different results using the same scope of study [7]. This study will focus on modelling of the growth of the national debt of Nigeria and mathematically analyze the impact of it and its various components on the health of the economy in general.

The main aim of this study is to model the growth and impact of external debt on the economic growth in Nigeria. The study is specifically set to:
i.   formulate a model of growth of the federal debt of Nigeria;
ii. mathematically analyze the impact of growth of the federal debt of Nigeria and its various components on the health of the economy in general;
iii. evaluate some economic policies for future years based on our model;
iv. determine the sensitivity of our model to various parameters and discuss any discrepancies with literature;
v.   determine long relationship between external debt and economic growth in Nigeria.

Nigeria's external debt relief
Several theoretical contributions have been made as regards the subject matter of external debt and economic growth. These theories are of relevance to this study as they serve as a building block to this research work and as such the following theories will be discussed; the dual-gap theory, debt overhang theory, crowding-out effect theory, dependency theory and the Solow-growth model.

II.   METHODS
The aim of this research study is to examine the growth and impact of external dept on the growth of the Nigerian economy. This chapter consists of mathematical model and theoretical framework which provides the more light into the study.

Growth Model
We will employ the Malthusian and Logistic growth models in our study.

Malthusian population model:
The Malthusian population model states that rate of change of population at time, t, is proportional to population at time, t

$$\frac{dN(t)}{dt} = rN(t) \qquad (1)$$

In other words, the relative rate of growth $\frac{1}{N(t)}\frac{dN(t)}{dt}$ is constant. The model (1) is solved by separation of variables to obtain the solution (2)

$$N(t) = N_0 e^{rt} \qquad (2)$$

Logistic growth model
In an environment that will support a limited population it is assumed that the rate of growth of population decreases as the limiting population is approached. An appropriate model (the Verhulst-Pearl model) is given by

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{N_m}\right) \quad (3)$$

N_m is the maximum population which can be sustained and r is the intrinsic growth rate. We can solve this equation for N, assuming $N = N_0$ at t = 0.

In order to avoid complexity due to fractional expression by setting

$x(t) = \frac{N(t)}{N_m}$ $\;such\,that\; N(t) = N_m x(t)$, then (3) becomes

$$N_m \frac{dx(t)}{dt} = rN_m x(t)\big(1 - x(t)\big)$$

$$\frac{dx(t)}{dt} = rx(t)\big(1 - x(t)\big) \quad (4)$$

Separating the variables in (3.4) we have

$$\frac{dx(t)}{x(t)(1 - x(t))} = rdt \qquad (5)$$

Splitting the left hand side of (5) into partial fractions, (5) becomes

$$\frac{dx(t)}{x(t)} + \frac{dx(t)}{\big(1 - x(t)\big)} = rdt$$

Integrating both sides we get

$$ln\left|\frac{x(t)}{(1 - x(t))}\right| = rt + c \qquad (6)$$

Applying boundary conditions we have

$t = 0,\; x(t) = x_0\left(= \frac{N_0}{N_m}\right)$ so that (6) becomes

$$ln \left| \frac{x(t)}{(1-x(t))} \cdot \frac{1-x_0}{x_0} \right| = rt \quad \Rightarrow \frac{x(t)}{(1-x(t))} \cdot \frac{1-x_0}{x_0} = e^{rt}$$

$$(7)$$

Solving for $x(t)$ we get

$$x(t) = \frac{e^{rt}}{\left(\frac{1}{x_0} + e^{rt} - 1 \right)}$$

Multiplying through by e^{-rt} we have

$$e^{-rt} x(t) = \frac{1}{\left(\frac{1}{x_0} + e^{rt} - 1 \right)} \quad \Rightarrow x(t) = \frac{1}{1 + \left(\frac{1}{x_0} - 1 \right) e^{-rt}}$$

$$(8)$$

Now putting $x(t) = \frac{N(t)}{N_m}$ $\quad and\ x_0 = \frac{N_0}{N_m}$ so that (9) becomes

$$\frac{N(t)}{N_m} = \frac{1}{1 + \left(\frac{N_0}{N_m} - 1 \right) e^{-rt}} \quad \Rightarrow N(t) = \frac{N_m}{1 + \left(\frac{N_0}{N_m} - 1 \right) e^{-rt}}$$

$$(9)$$

Exponential Family and Regression
Definition 1: Exponential function
The equation $f(x) = b^x$ with b>0, b≠1 defines an exponential function. b is the base; the domain is $\Re$, all real numbers, and the range is the set of all positive real numbers.Sometimes linear regression can be used with relationships which are not inherently linear, but can be made to be linear after a transformation. In particular, we consider the following exponential model:

$$y = \alpha e^{\beta x} \qquad (10)$$

Taking the natural log (see Exponentials and Logs) of both sides of the equation, we have the following equivalent equation:

$$ln\ y = ln\ \alpha + \beta x \qquad (11)$$

This equation has the form of a linear regression model (where we have added an error term ε):

$$y^{'} = \alpha^{'} + \beta x + \varepsilon \qquad (12)$$

It can be observed that, since $\alpha e \beta^{(x+1)} = \alpha e^{\beta x} \cdot e^{\beta}$, we note that an increase in x of 1 unit results in y being multiplied by e^{β}. A model of the form

$$ln\ y = \beta x + \delta \qquad (13)$$

is referred to as a log-level regression model. Clearly any such model can be expressed as an exponential regression model of form y = $\alpha e^{\beta x}$ by setting α = e^{δ}.

Exponentials and logs
For any number b and positive integer n, we define exponentiation, i.e. b raised to the power n, as follows: $b^n = b \cdots b = b$ multiplied by itself n times
We can extend this definition to non-positive integers n as follows:

$$b^n = \begin{cases} b \cdots b & if\ n > 0 \\ b^{-n} & if\ n < 0 \\ 1 & if\ n = 0 \end{cases} \qquad (14)$$

For example, $2^3 = 2 \cdot 2 \cdot 2 = 8$, $2^{-3} = 1/8$ and $2^0 = 1$
Exponentiation has the following properties:

$$b^m \cdot b^n = b^{m+n} (b^m)^n = b^{mn} (15)$$

where n > 0, we can also define $b^{1/n} = \sqrt[n]{b}$ the number a such that a multiplied by itself n times is b. We can extend this definition to

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b} \right)^m \quad (16)$$

where m and n are any integers. Without getting into all the details, b^a is defined for any a, and can be calculated in Excel by b^a. The properties noted above for integer exponents can be extended to any exponents, namely

$$b^a \cdot b^c = b^{a+c}, \quad (b^a)^c = b^{ac}, \quad b^{-a} = \frac{1}{b^a} (17)$$

$\log_b a$, called the log of a (base b) = the number c such that b^c = a. Thus, the log function is the inverse of exponentiation and has the following properties:

In this website we use logs with base = 10 (called log base 10 and written simply as log a) and logs with base e where e is a special constant equal to 2.718282…. The log of a base e is called the natural log of a and is written as ln a.

Formulation of Model of Growth of External Debt Model
Assumptions of the formulation of the debt model
The following assumptions were made in the formulation of the model;
i.   We assume a Keynesian economy.
ii.   The only source of governmental income is oil.
iii. Linear dependency exists between the rate of change of external debt and the difference between federal income and expenditures.
iv.   The government will always borrow if its federal expenditures are greater than federal income.
v.   All form of government revenue is represented by T(t)
vi.   All form of government expenditure is represented F(t)
vii. Other assumptions are listed as needed during the modelling process.

Parameters and variables of the Model
As established earlier, the following parameters will enable us establish our debt model

Table 1: Table showing parameters and variables of the model

Parameter/ Variable	Meaning
N (t)	External Debt
T (t)	Revenue generated
F (t)	Federal expenditures
G (t)	Gross domestic product
t	Time
k	Proportionality Constant
E(t)	Aggregate Expenditures
C(t)	Consumption Expenditures
I(t)	Investment Expenditures
M(t)	Imports
X(t)	Exports
Nx(t)	Net Exports
Y(t)	Disposable Income
MPC	Marginal Propensity to Consume

Flow diagram for the growth model of external debt

We begin our model by asserting the idea that the rate of change of external debt is proportional to the difference between federal income and federal expenditures. Assuming that federal income is equal to the tax revenue of the government, and that the rate of debt change is linearly proportional to this difference between income and expenditure, we can conclude as summarized in the model flow diagram below:
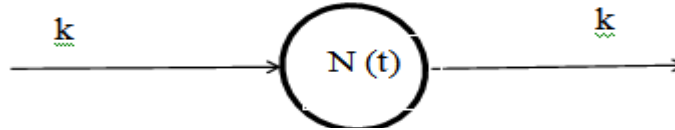

Figure 1: A model flow diagram between three states

Therefore, the transition debt growth model following from Figure 1 above is given by

$$\frac{dN(t)}{dt} = kN(t) \qquad (18a)$$

Where $N(t) = -T(t) - \left(-F(t)\right)$ which yields

$$\frac{dN(t)}{dt} = -k[T(t) - F(t)] \qquad (3.18b)$$

The above relationship basically summarizes the idea that the rate of change of national debt is inversely proportional to the difference between federal income and federal expenditures. We now need to derive functions for the federal income, T(t) and federal expenditures F(t). We begin by extracting data for these two quantities from the analysis of the data sources in Appendix A as found in Appendices B and C respectively. Our first attempt to deriving these functions is to empirically model historical data using exponential regression. Considering the table on Coefficients [a] in Appendix B and C we write the regression equations (19) and (20) respectively;

$$ln\big(T(t)\big) = 22.335 + 0.245t \qquad (19)$$

$$ln\big(T(t)\big) = 22.667 + 0.216t \qquad (20)$$

Substituting equations (19) and (20) into (18b) and integrating with respect to t we have the Nigeria's national debt to be given as

$$N(t) =$$
$$-k\left[(5.01 \times 10^9)\frac{\left(\frac{1438473717286851}{1125899906842624}\right)^{t}}{log\left(\frac{1438473717286851}{1125899906842624}\right)} - (7.05 \times 10^9\frac{1397357052898259}{1125899906842624}log1397357052898259}{1125899906842624t}\right]$$
$$(21)$$

$$with$$

$$T(t) = (5.011 \times 10^9)\frac{\left(\frac{1438473717286851}{1125899906842624}\right)^{t}}{log\left(\frac{1438473717286851}{1125899906842624}\right)} \qquad (22)$$

$$F(t) = (7.054 \times 10^9)\frac{\left(\frac{1397357052898259}{1125899906842624}\right)^{t}}{log\left(\frac{1397357052898259}{1125899906842624}\right)} \qquad (23)$$

Formulation of Model of Impact of External Debt

In the formulation of the model of impact of national debt, we begin to explore the impact of

the national debt of Nigeria on its national economic infrastructure by asserting the fact that
$$E(t) = C(t) + I(t) + F(t) + N(t)X(t) \qquad (24)$$
We also assume a Keynesian economy, where Say's law that demand would always equal supply in an economy does not hold and consumer spending is proportional to income by a propensity of consumption. Since the Gross DomesticProduct of a nation can be represented by its income, and consumer spending is equivalent to the aggregate expenditures of an economy, we can assert that
$$E(t) = E(0) + MPC\big(G(t)\big) \qquad (25)$$
We can now formulate an expression for the marginal propensity to consume as
$$MPC = \frac{dE(t)}{dG(t)} \qquad (26)$$
We are now interested in the break-even point of the expediture-GDP graph; noting the fact that the break-even point, were all earnings are totally spent, is located at the intersection of a fourty five degree ray from the origin and the economy's expenditure curve, namely of the following two equations,
$$E(t) = E(0) + MPC\big(G(t)\big) \qquad (27)$$
$$E(t) = G(t)$$
we can quantify the break-even point to be
$$B = \frac{E(0)}{1 - MPC} = \frac{E(0)}{1 - \frac{dE(t)}{dG(t)}} \qquad (28)$$
On the basis of this idea, we can now redefine our original model of rate of change of external debt to be:
$$\frac{dN(t)}{dt} = \begin{cases} -k[T(t) - F(t)], & G(T) < \frac{E(0)}{1 - \frac{dE(t)}{dG(t)}} \\[2ex] k[T(t) - F(t)] & G(T) > \frac{E(0)}{1 - \frac{dE(t)}{dG(t)}} \end{cases} \qquad (29)$$
because below the break-even point, the infrastructure of the economy is in the mode of saving, and most likely, paying of its previous debt in order to maintain a balanced budget.

Likewise, at a GDP greater than the break-even point, the economy is dis-saving and either using of its previous savings to maintain an expenditure below current income, or borrowing externally for that purpose. Note that
$$\frac{dG}{dt} = \left(\frac{1}{1 - MPC}\right) \cdot \frac{dE}{dt} = \left(\frac{1}{1 - MPC}\right) \cdot \frac{dE}{dt} \qquad (30)$$

Model Analysis
Solving the model by separation of variables we obtain
$$N(t) = K[exp(F - T)t] \qquad (31)$$
K is the initial debt size
Let $F - T = Q$, then equation (26) becomes
$$N(t) = K[exp(Q)t] \qquad (32)$$

Equation (29) can be used to describe the growth rate of Nigeria debt profile from year to year and its effect in the long run based on the critical cases considered here:

Case I: At steady state, $\frac{dN(t)}{dt} = -k[T(t) - Ft = 0$ which show that, $Q = 0$
$\Rightarrow T(t) - F(t) = 0 \Rightarrow T(t) = F(t)$. Here the external debt is steady and constant.

Case II: When $Q < 0$, it follows that $-k[T(t) - Ft < 0$ such that
$-kT(t) + kF(t) < 0 \Rightarrow F(t) < T(t)$, then after a very long time,
$$\lim_{t \to \infty} N(t) = \lim_{t \to \infty} K[exp(F - T)t] \to 0$$
Signifying that there will be no debt after a long time.

Case III: When $Q > 0$, it follows that $-k[T(t) - Ft > 0$ such that
$-kT(t) + kF(t) > 0 \Rightarrow F(t) > T(t)$, then after a very long time,
$$\lim_{t \to \infty} N(t) = \lim_{t \to \infty} K[exp(F - T)t] \to \infty$$
Signifying that there external debt will grow beyond bond after a long period of time.

III.   RESULTS
The result of the study are presented on Table 1 - Table 2 and with the aid of graphs presented on Figure 1 – Figure 10 showing the growth and impact of the Nigeria's national debt. The data points for the available data used for the study are plotted in Figures 11 – 13.

The solution obtained with aid of symbolic (syms) computation using MATLAB 7.5 is given by
$$T(t) = (5.011494445 \times 10^9)^{\frac{\left(\frac{1438473717286851}{1125899906842624}\right)^t}{log\left(\frac{1438473717286851}{1125899906842624}\right)}}$$
$$(4.1)$$
$$F(t) = (7.054982911 \times 10^9)^{\frac{\left(\frac{1397357052898259}{1125899906842624}\right)^t}{log\left(\frac{1397357052898259}{1125899906842624}\right)}}$$
$$(4.2)$$
Then, the Nigeria's national debt is given by
$$N(t) = -k\left[(5.011494445 \times 10^9)\frac{\left(\frac{1438473717286851}{1125899906842624}\right)^t}{log\left(\frac{1438473717286851}{1125899906842624}\right)} - (7.054982911 \times 10^9)\frac{\left(\frac{1397357052898259}{1125899906842624}\right)^t}{log\left(\frac{1397357052898259}{112589990684262\ 4}\right)}\right]$$
$$(4.3)$$

Descriptive Statistics	N	Min.	Max.	Mean	Std. Dev.	Variance
Year	34	1981	2014	1997.50	9.958	99.167
Oil Revenue	34	7	8879	2149.04	2779.531	7725791.769
Government Expenditure	34	10	5185	1316.28	1679.951	2822235.517
Valid (listwise)	N 34					

Table 2: Summary of Federal Government FiscalOperation 1995 to 2000

Table 3: Table of values showing the Government Income, T(t), Expenditure, F(t) andNational Debt, N(t) from 1980 to 1997

Time, t [years]	Study Period [Years]	Govt. Income, $T(t)$ x 10^{12}	Govt. Expenditure, $F(t)$ x 10^{12}	National Debt, $N(t)$ x 10^{12} for k = 0.01	National Debt, $N(t)$ x 10^{12} for k = 0.02	National Debt, $N(t)$ for k = 0.03
0	1980	0.0205	0.0327	0.0001	0.0002	0.0004
1	1981	0.0261	0.0405	0.0001	0.0003	0.0004
2	1982	0.0334	0.0503	0.0002	0.0003	0.0005
3	1983	0.0427	0.0624	0.0002	0.0004	0.0006
4	1984	0.0545	0.0775	0.0002	0.0005	0.0007
5	1985	0.0696	0.0962	0.0003	0.0005	0.0008
6	1986	0.0890	0.1194	0.0003	0.0006	0.0009
7	1987	0.1137	0.1481	0.0003	0.0007	0.0010
8	1988	0.1452	0.1839	0.0004	0.0008	0.0012
9	1989	0.1855	0.2282	0.0004	0.0009	0.0013
10	1990	0.2370	0.2832	0.0005	0.0009	0.0014
11	1991	0.3028	0.3515	0.0005	0.0010	0.0015
12	1992	0.3869	0.4362	0.0005	0.0010	0.0015
13	1993	0.4943	0.5414	0.0005	0.0009	0.0014
14	1994	0.6316	0.6720	0.0004	0.0008	0.0012
15	1995	0.8069	0.8340	0.0003	0.0005	0.0008
16	1996	1.0309	1.0351	0.0000	0.0001	0.0001
17	1997	1.3172	1.2846	-0.0003	-0.0007	-0.0010

Table 4: Table of values showing the Government Income, T(t), Expenditure, F(t) and National Debt, N(t), from 1997 to 2014.

Time, t [years]	Study Period [Years]	Govt. Income, $T(t)$ x 10^{13}	Govt. Expenditure, $F(t)$ x 10^{13}	National Debt, $N(t)$ x 10^{13} for k = 0.01	National Debt, $N(t)$ x 10^{13} for k = 0.02	National Debt, $N(t)$ x 10^{13} for k = 0.03
17	1997	0.1317	0.1285	-0.0000	-0.0001	-0.0001
18	1998	0.1683	0.1594	-0.0001	-0.0002	-0.0003
19	1999	0.2150	0.1979	-0.0002	-0.0003	-0.0005
20	2000	0.2747	0.2456	-0.0003	-0.0006	-0.0009

21	2001	0.3510	0.3048	-0.0005	-0.0009	-0.0014
22	2002	0.4484	0.3783	-0.0007	-0.0014	-0.0021
23	2003	0.5729	0.4695	-0.0010	-0.0021	-0.0031
24	2004	0.7319	0.5827	-0.0015	-0.0030	-0.0045
25	2005	0.9351	0.7232	-0.0021	-0.0042	-0.0064
26	2006	1.1947	0.8975	-0.0030	-0.0059	-0.0089
27	2007	1.5264	1.1139	-0.0041	-0.0082	-0.0124
28	2008	1.9501	1.3825	-0.0057	-0.0114	-0.017
29	2009	2.4915	1.7158	-0.0078	-0.0155	-0.0233
30	2010	3.1832	2.1295	-0.0105	-0.0211	-0.0316
31	2011	4.0669	2.6429	-0.0142	-0.0285	-0.0427
32	2012	5.1960	3.2801	-0.0192	-0.0383	-0.0575
33	2013	6.6385	4.0709	-0.0257	-0.0514	-0.077
34	2014	8.4815	5.0524	-0.0343	-0.0686	-0.1029
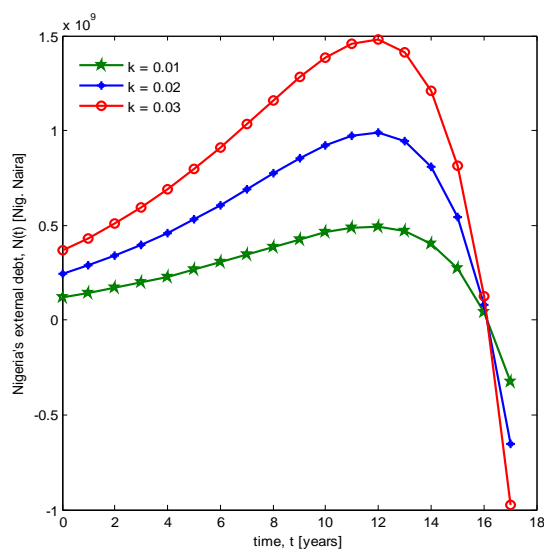
Figure 2: A graph showing the variation in Nigeria's external debt for $k = \{0.01, 0.02, 0.03\}$ time, $t \in [0, \ 17]$ here correspond to the study period between 1980 to 1997
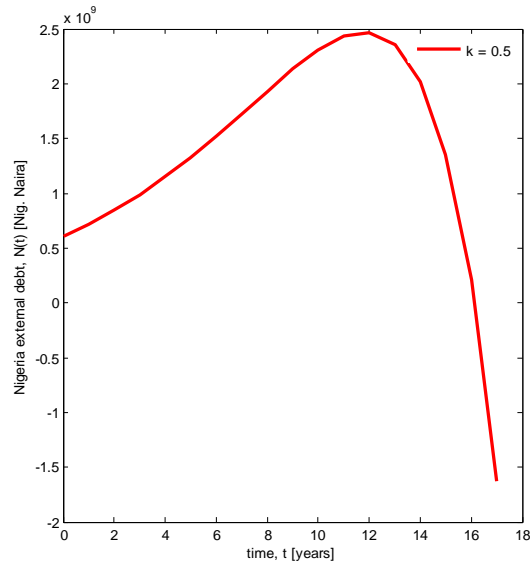
Figure 3: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [0, \ 17]$ which is the study period between 1980 to 1997
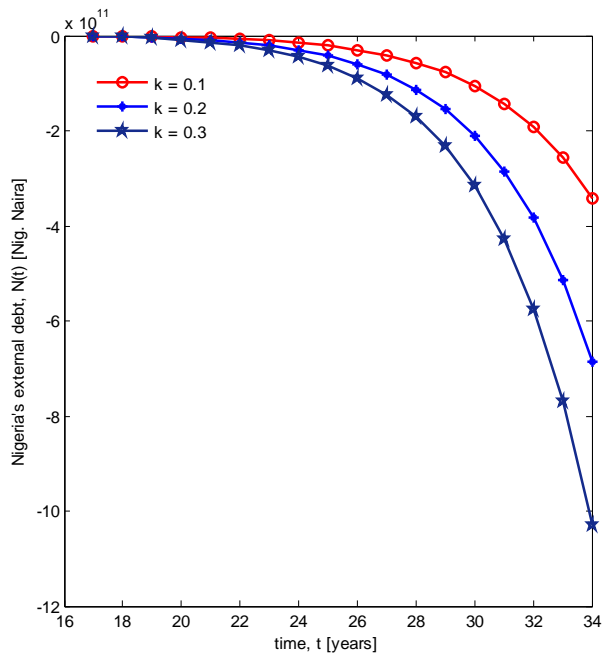
Figure 4: A graph showing the variation in Nigeria's external debt for$k = \{0.01, 0.02, 0.03\}$at time, $t \in [16, \ 34]$ which is the study period between 1997 and 2014.
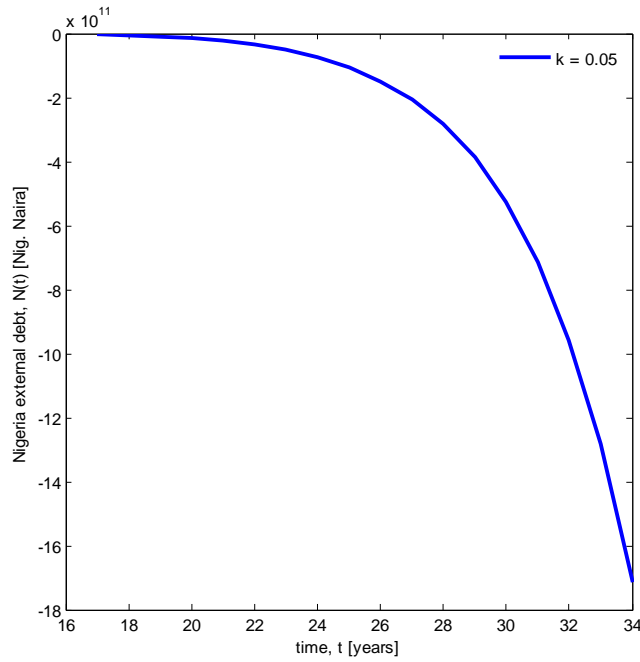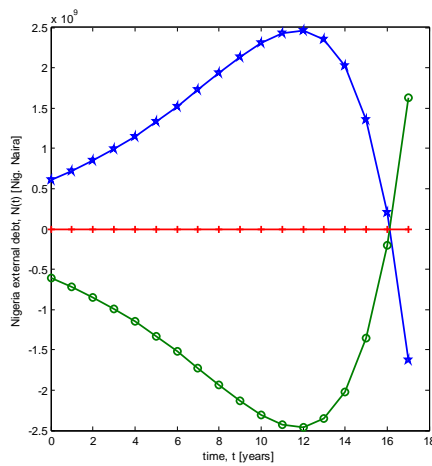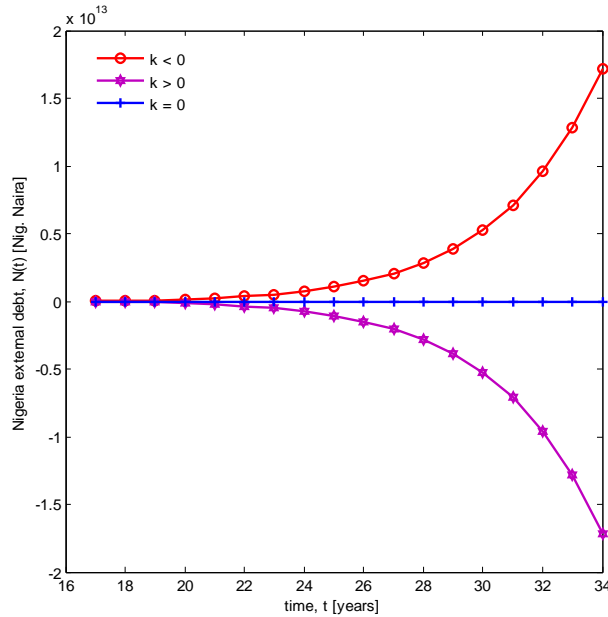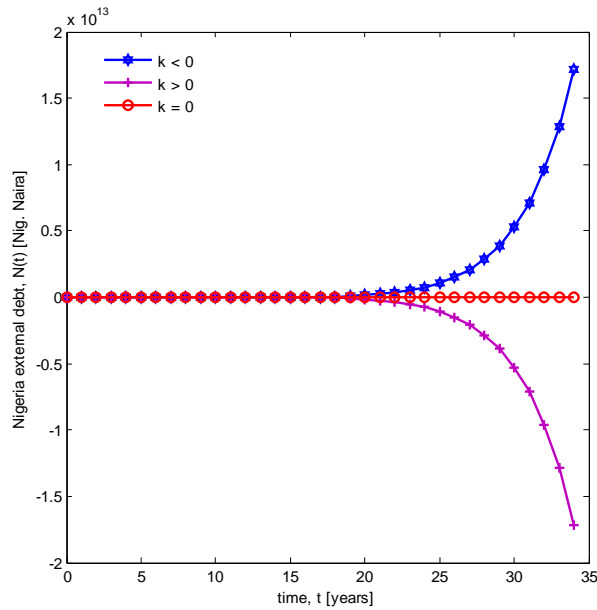
Figure 5: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [0, \ 17]$ which is the study period between 1997 to 2014.

Figure 6: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [0, \ 17]$ which is the study period between 1980 to 1996

Figure 7: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [17, 34]$ which is the study period between 1980 and 2014.

Figure 8: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [0, 34]$ which is the study period between 1980 and 2014.
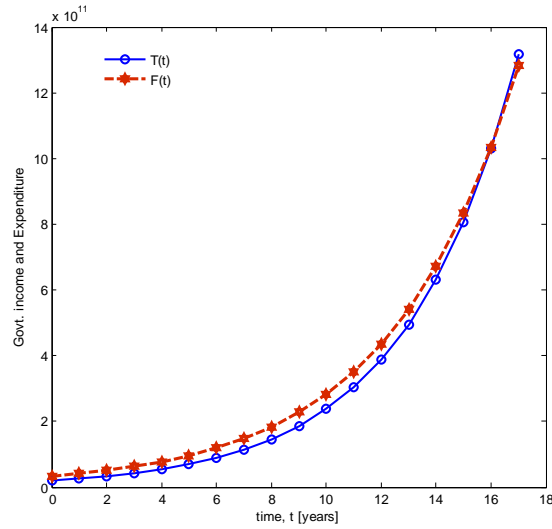
Figure 9: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [0, 17]$ which is the study period between 1980 and 1996
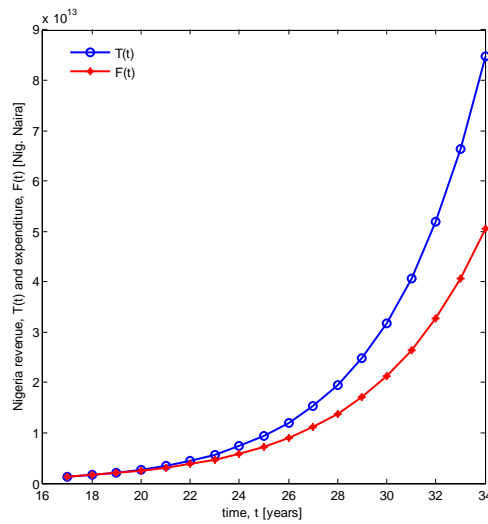

Figure 10: A graph showing the variation in Nigeria's external debt for $k = 0.5$ at time, $t \in [17, 34]$ which is the study period between 1997 and 2014.
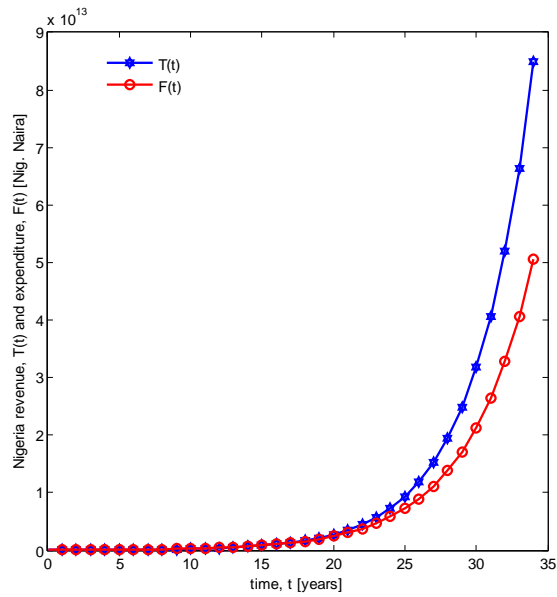
Figure 11: A graph showing the variation in Nigeria's external debt for $k = 0.5$ **at time,** $t \in [0, 34]$ **which is the study period between 1980 and 2014.**
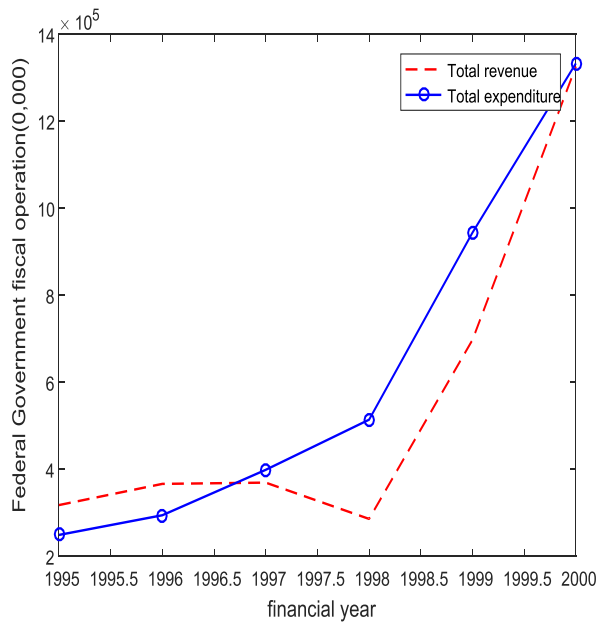

Figure 12: Graph of summary of Federal Government fiscal Operation from Table 1.
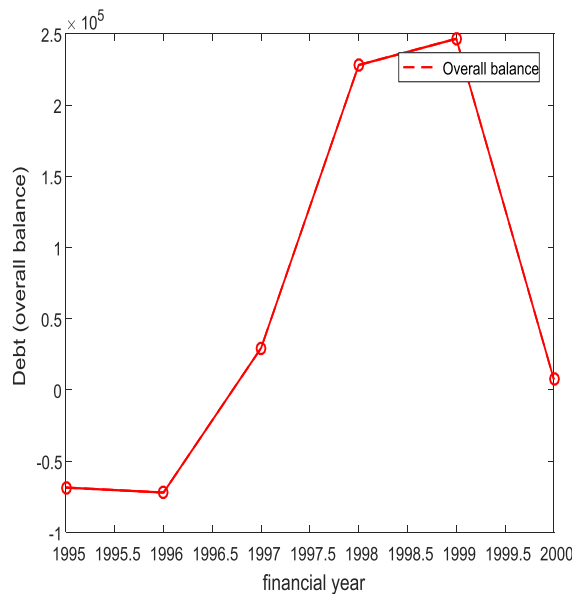
Figure 13: Graph of overall balance of Federal Government fiscaloperation from table 1.
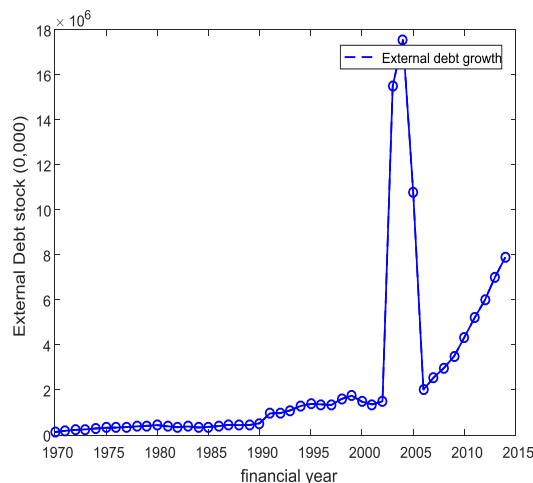

Figure 14: Graph of external Debt stocks of Nigeria from Table 3.

IV. DISCUSSION

It is observed from Figure 1 that Nigeria's national debt, N(t) increased in the first twelve (12) years (i.e. 1981 – 1992) of the study and continued to decrease for all the values of k > 0. it is also evident that N(t) increases as k increases (i.e. N(t) = 5.0 x 10^8, 1.0 x 10^9, 1.5 x 10^9 for k = 0.01, 0.02 and 0.03 respectively). In order to understand and be convinced of this trend, we obtain N(t) for k = 0.5 as shown on Figure 2 .

The Nigeria's national debt, N(t) as presented in Figure 2 increases from 6.0 x 10^8 in the year 1980 to 2.5 x 10^8 in the year 1992 for k = 0.5 and continue to decrease with time. These changes are informed by the available economic policies of the nation. It is believed that factors

that could lead to the growth in Nigeria's external debt could be both social and economic issues which include corruption, poor infrastructure, inflation and worth of the naira, to mention but a few.   If these factors are watched and well managed, the national debt profile will change in the interest of the citizens as well as the inhabitants of the nation.

This trend is in analogy with Iyoha (2000) who presented that in the face of the declining trend in world oil prices, Nigerian government has been facing difficulties in financing deficit budgets and developmental projects. This led to borrowing from external sources such as World Bank, International Monetary Fund (IMF) and Paris club amongst others. The responsibility of managing

external debt among other issues led to the establishment of other departments in the Central Bank of Nigeria (CBN) to undertake the functions in collaboration with the Federal Ministry of Finance (FMF) and other relevant agencies. Since 1980s, Nigeria has devised several debt management policies in order to make the debt–service burden bearable and avoid defaulting.

External debt service on the other hand, reduces public investment funds, employment and private income. It also reduces the country's currency and compound balance of payment problems. In view of these, several policies were introduced to reduce the magnitude of public debt, ameliorate the debt service burden in order to stimulate sustained growth in the Nigerian economy. But, Henry observed that these policies have not been able to restrain the growth of foreign debt [19].

The negative turn in the National debt, N(t) of Nigeria continues as presented in Figure 3. It is observed that at the thirty fourth year of the study (i.e. in the year 2014) as k varies across $k = \{0.01, 0.02, 0.03\}$, the Nigeria's national debt, $N(t) =$
$-3,430,000,000,000.00, -686,000,000,000.00,$
$1,029,000,000,000.00$ respectively showing that the Nigeria's external debt increases with time across increasing values of k. A similar trend is observed for k = 0.5 and presented in Figure 4.

The justification for the trend shown in Figure 4 is studied and reporteddue to the fact that; debt conversion programme was made to complement other strategies of debt management where the Nigeria's debt conversion exercise involved the sale of an external debt instrument for a domestic debt or equity participation in domestic enterprises. On external front, Nigeria's credit worthiness increased, thereby, making the economy worthy to access shortand medium term credits which enhance net capital inflows necessary for employment and growth. In the domestic economy, potential new export earnings and gains from new investment as well as the money budgeted for debt servicing are expected to be channeled into growth enhancing projects- as this result to rises in investment, employment and output.

Since Nigeria's external debt is affected by the value of k, it is presented in Figure 5, three special cases involving k such that for any k > 0, the Nigeria's national external debt increased to a peak before a downturn in the debt profile is experienced after the 12th year (1992) of the study. This decrease also continued and aligned with the case where k < 0 and k = 0. The case of k = 0 is

an equilibrium point where the country will experience debt free-state.

Figure 8 is a graph showing the variation in income and revenue for k = 0.5 within the study period, t ∈ [0, 17] which is the study period between the years; 1980 and 1997. At every point in time, the Government expenditure outweighed her income. There was a slight balance at t = 16 years (i.e. in the year 1996) and the revenue is slightly above the expenditure in the year 1997.

From 1997 down to 2014, the revenue continued to be higher than the government expenditure as shown in Figures 9 and 10 which could be for the fact that factors that raise the expenditure were minimized to achieve greater revenue.

Considering the raw data plotted in Figure 11, there is a clear increase in expenditure signifying that the federal government has spent more than she has earned within the period under study. The figure also shows that government expenditure responded to changes in total revenue. Figure 10 provides some preliminary analyses involving the description of relevant statistical properties of the variables in Figure 11. These indicate increase in the debt profile of the government as expenditure is more than revenue.

Figure 13 indicates steady increment of the Nigerian Debt even after the restructuring arrangements with Paris Club of Creditors in 1986, 1986, 1991 and 2000. This did not stop the ''leaps'' and ''jump'' in the external debt stock even after 2005 debt relief. This same graph showed the latest value for external debt stock in Nigeria which was $ 26,858,200 as of 2014. Over the past 44 years, the value for this indicator has fluctuated between $ 17,560,270,000 in 2004 and $ 130,891,000 in 1970. The above result indicates that if we don't review our policies with respect to expenditure the debt will always be on the increase to the detriment of the economy and the country at large. With this kind of trend, it is evident that revenue is less than the expenditure aggregately, thus giving room to external debt.

Following from the model analysis, 3.4, the aggregate expenditures model is used as a framework for determining GDP in the economy. All the earnings are totally spent. Below the break-even point, the infrastructure of the economy is in the mode of saving and most likely paying off its previous debt in order to maintain a balanced budget. Likewise, at a GDP greater than the break-even point, the economy is dis-saving and either using off its previous saving to maintain expenditure below current income or borrowing externally for that purpose.

V.   CONCLUSION

The study examined the relationship between external debt and economic growth in Nigeria from 1981 to 2014. The study used descriptive, econometric and mathematical tools. The model was formulated and analysed using the theory of ordinary differential equations in the form of exponential growth model such that each of the variables and the result showed that all the variables were integrated at first difference. The study revealed positive impact of external debt on economic growth in Nigeria in the long-run only. However, external debt servicing had both long-run and short-run negative effect on economic growth in Nigeria.Growth in capital expenditure, growth rate of exports, and external debt stock had positive impact on economic growth while external debt servicing and interest rate were inversely related to the growth of the economy**.**

It was also observed that the question of positive contribution outweighing the negative contribution is not the subject of this work. Nigeria's external debt profile continues to rise as time goes on. It is a clear fact from observable reality that ineffective utililization of debt will make repayment a difficult task, as such the interest will keep accruing (a time almost to be the tune of the capital), then repayment becomes a problem and such debt will become a bad debt.

VI.   RECOMMENDATIONS

Based on the results, the study made the following recommendations;
i.   The modalities of incurring external debt and their application should be technically and tactically analyzed prior to accessing the debt.
ii.   Although, external debts are meant to boost the economic growth and development of the debtor country and the standard of living of the citizenry, as such Nigeria government should ensure that debts incurred are channel towards productive uses and sourcing external debt to be considered as a means of long run development and not just solving short run problem.
iii.   External debt loans should be used only for productive investment of highest priorities that would help in yielding returns. This would help in deriving enough money for external debt servicing; otherwise, the huge external debt would throw the economy into series and "serious" economic problems.
iv.   Nigeria should use her accumulated external foreign reserves instead of incurring more external debts, as this will ensure increase in real economic growth and reduce capital flights through repayments of debts to external sources.
v.   Instead of continuous debt servicing, there should be an arrangement for repayment of the loan(s) within the agreeable time. This will curtail the associated leakages from the economy.

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